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AUTONOMOUS MOTION PLANNING FOR SPACECRAFTS NEAR SMALL SOLAR SYSTEM BODIES: SIMULTANEOUSLY REFINING THE GRAVITATIONAL FIELD MODEL AND RE-PLANNING GRAVITY DEPENDENT MANEUVERS

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Abstract

Dynamic path planning near small celestial bodies has the potential to improve asteroid study, landing strategies, and scouting for in-situ resources, as well as autonomous missions for comet interception and deep space exploration. Strategic missions to orbit celestial bodies have primarily considered spacecraft trajectories as a two step process: capture of the spacecraft within the gravitational influence of the body, followed by in-orbit maneuvers. While maneuver planning in the gravitational field of larger solar system bodies (with near-uniform gravity fields) is relatively straightforward, planning similar maneuvers around smaller bodies such as asteroids and comets is more challenging. Moreover, *a-priori* maneuver planning approaches that use earth-based measurements will tend to generate motion plans that have a monolithic profile. Fine grained motion plans that respond to mission conditions require a detailed understanding of the gravitational forces around the body---which can only be obtained once a craft is in orbit, assuming the craft has sufficient onboard sensors. For example, the gravity model can be analyzed to provide information about the mass, density, and material distribution across the body. We propose a method for autonomous motion planning around small bodies that continually refines the gravitational model of the body while simultaneously using the model to perform more and more accurate orbital maneuvers.

Our research focuses on a problem variant where the orbital maneuvers are designed to refine the gravity map as quickly as possible. However, the basic idea of simultaneous gravity model refinement and motion planning is relevant to a variety of space exploration and scientific missions. We use a receding horizon approach. During each planning epoch, the planner considers the gravitational influence over a tree of orbital maneuver sequences (between discrete points around the celestial body). Starting with the (low fidelity) gravity model created from earth-based observations, the gravity model is continually updated during the mission as the spacecraft experiences varying gravitational forces. Onboard instruments measure the force observed by the craft and the gravity model is updated with each maneuver, eventually providing a high fidelity gravity field model of the body. The updated model is simultaneously and continually used to re-plan the craft's trajectory during the mission, ensuring that each maneuver respects the most up-to-date model of the body's gravity field (that is, respects the gravity data observed during the mission so far). Such an approach has the potential to expand to autonomous spacecraft missions to perform maneuvers near small celestial bodies

Keywords: *Gravity, spacecraft, maneuvers, motion planning, dynamic paths, space robotics.*

Nomenclature

a	Semi major axis
C	Configuration space
C_{free}	Free space
C_{obs}	Observed space
D	Spacecraft Dynamics
dv	edge length of tree branch
F	Force exerted
G	Gravitational constant
G_a	Gravitational Acceleration
G_{gt}	Ground truth gravity
G_f	On board gravitational field model
H	Heuristics
i	Current iteration
J	Cost function
L	Waypoint List
M	Motion Planner
m_1	Mass of body 1
m_2	Mass of body 2
N	Total iterations
o_{init}	Initial waypoint
o_{goal}	Goal waypoint
o_{near}	Nearest waypoint
o_{opt}	Viable waypoint
o_{rand}	Randomly sampled waypoint
o_{exp}	Explored waypoint
o_{obs}	Observed waypoint
r	Distance between 2 bodies
R	Reference gravity model
t	time horizon
t_0	Initial time of exploration
Δt	Time taken to explore next waypoint
$< T >$	Trajectory
v	Velocity
V	Gravitational potential
w_u	Weight of control nodes
$\wedge$	Objective function
λ	Position of the spacecraft in cartesian space

Acronyms/Abbreviations

AU	Astronomical Unit
BVP	Boundary Value Problem
CPU	Central Processing Unit
DoF	Degrees of Freedom
EGM	Earth Gravity Model
GFM	Gravity Field Model
GGM	GRACE Gravity Model
JGM	Joint Gravity Model
NEAR	Near Earth Asteroid Rendezvous
PRM	Probabilistic Road Map
RDT	Rapidly Exploring Dense Trees
RRT	Rapidly Exploring Random Trees
SLAM	Simultaneous localisation and mapping
SOI	Sphere of Influence
SSSB	Small solar system body

1. Introduction

The presence of small solar system bodies (SSSB) has sparked human curiosity to learn about their creation, their trajectories, and the resources they house. In near-present times, these bodies are almost exclusively studied by remote sensing observations. Even the limited number of flyby and impact missions that have been performed have relied heavily on earth-based measurements. These observations include orbital properties like trajectory and velocity; physical properties like albedo, density and gravitational influence.

The knowledge of the gravitational fields of these bodies can be used to estimate their structure. The gravity field model of a SSSB can relay information like mass-density distribution and in some cases, the material composition of the body. An accurate gravitational field model is also essential for enabling precise flyby and landing missions.

Owing to their relatively small mass and irregular shape and composition, SSSBs exert gravitational forces that are magnitudes smaller and more spatially varied as compared to larger solar system bodies. As a result, earth-based measurements generate low fidelity models that may not accommodate the smaller gravitational forces exerted by a SSSB.

To obtain detailed field models, measurements must be made in the vicinity of an SSSB, an act that necessitates the use of precise in-orbit maneuvers. However, the erratic gravitational forces of attraction experienced by a spacecraft in the vicinity of an SSSB makes collecting such in-orbit measurements challenging. A potential solution is to continually refine a gravity model that becomes more and more accurate as more and more measurements are taken. From a motion planner's point of view, replanning based on an ever improving gravity field model is equivalent to replanning within an environment that is dynamic with respect to gravitational forces. Even though the gravity of the SSSB may not be changing, the gravity model used by the motion planner changes each time it is refined based on additional data.

Our research focuses on performing near-real time path planning for orbital maneuvers in the dynamic gravity field of SSSBs. The trajectories are planned with the intention of refining the gravity field map as quickly as possible with the goal of achieving a stable orbital motion(see Fig 09 for concept). We use a receding horizon approach. During each planning epoch, the planner considers the gravitational influence over a tree of orbital maneuver sequences (between discrete points in the orbit). Way-points are explored over sparsely-expanding geometric random-trees. Starting with the (low fidelity) gravity model created from earth-based observations, the gravity model is continually updated during the mission(as seen in Figure 01 as the spacecraft experiences varying gravitational forces). On-board instruments measure the forces experienced by the craft, eventually providing a high-fidelity gravity field model of the body. The updated model is simultaneously and continually used to re-plan the craft's trajectory during the mission, ensuring that each maneuver respects the most up-to-date model of the body's gravity field. Such an approach has the potential to contribute to autonomous spacecraft maneuvers, flybys and surface landings missions.

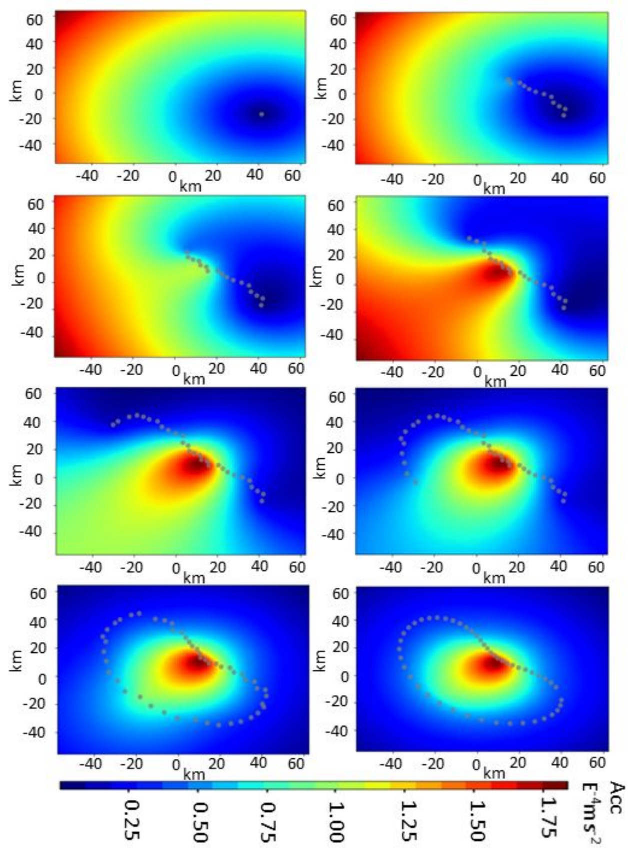

Figure 01: Refining onboard gravity model on the spacecraft. As the spacecraft executes its motion around the SSSB, the onboard gravity model (low fidelity model-represented by the first plot) gets refined with the forces experienced by the spacecraft. The blue region represents lower intensity attractive forces and the red region represents the higher intensity forces. The grey points are the spacecraft trajectory waypoints around the SSSB which become more stable as the gravity field is refined; the forces are updated with every epoch. The model gets more refined in intermediate plots and a higher fidelity model is obtained in the final plot. Here, the SSSB is located at the origin (0,0).

1.1. Background and Related Work

Earth-based experiments to measure gravity fields of other celestial bodies are often performed by radiometric instruments that measure the dissemination of the material surface of the bodies along with the rate of change position. Additionally, the gravitational effect on other nearby objects like moons and smaller rocks is also an important field of study. Missions like Near Earth Asteroid Rendezvous(NEAR) [1] and the Hayabusa provide insights into the gravity field as the spacecraft experienced forces, within the field of influence. These missions offer higher fidelity data than earth-based observations.

1.1.1. Sampling-based Motion planning

Sampling-based algorithms like PRM demonstrated by Kavarki [2] and RRT introduced by LaVelle [3] develop paths over way-points in the configuration space. RRT often returns a quick solution; however, the solution returned is only guaranteed to be feasible and not optimal[1].

In 2016, Li Y et al. worked on sparse RRT [4] that explored kino-dynamic path planning in configuration space for a near optimal solution. The work was based on RRT*[5] and RRT# motion planning approaches (see Figure 02).

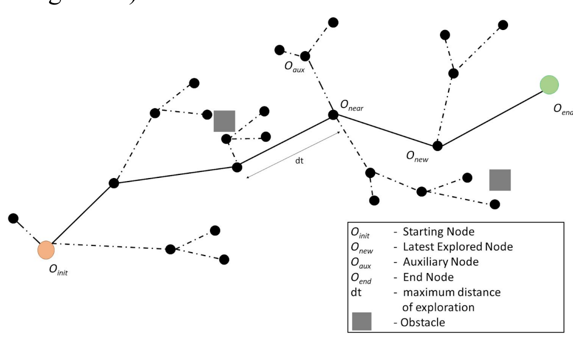

Figure 02: Concept of randomly exploring random trees Star (RRT*)[3]

Recent works are based on planners for system dynamics that can be linearly approximated[6]. Asymptotic near optimal motion planning in dynamic environments was progressed by Zakary et al.[7] as a stable sparse RRT that explored only a sparse space(as shown in figure 03) while exploring a sample space. The method introduced near real time path planning for unknown environments. The motion plan used forward propagation with a sparse data structure to answer path queries and generate trajectories.

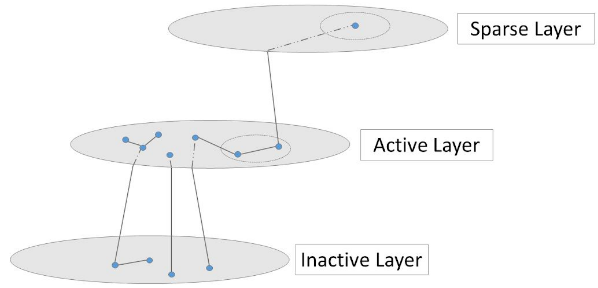

Figure 03: Stable sparse RRT for optimal path planning with asymptotic near optimality[7]

Research conducted by Otte and Frazzoli [8] studied the problem of replanning in a dynamic environment with unpredictably changing obstacles. In this work, an asymptotically optimal single-query algorithm was used to solve the dynamic motion planning problem (replanning the planned maneuver sequence after each time that unpredictably moving obstacles change the topology of the environment). The vast majority of previous work, including [8], has considered problem variants that deal with geometric changes related to obstacles. In contrast, in the current paper we consider replanning based on the changing gravity field model.

Based on asymptotically near-optimal approach for a kino-dynamic motion planning problem developed for a cost function J with finite cost J^*, the probability that the algorithm will find a solution of cost $J < tJ^*$ for some factor $t \geq 1$ converges to 1 almost surely as the number of iterations approaches infinity.

1.1.2. Motion Planning for spacecrafts

The notion of satellite autonomy as explored by Golden [9] states the requirement of a dexterous motion plan in space using state-of-the-art technological readiness. In 2002, Richards et al. [10] worked on trajectory planning using mixed-integer linear programming for satellite maneuvering. In 2007, Dario and Lorenzo[11] introduced autonomous and distributed motion planning for satellites in a swarm ecosystem by inverse dynamic calculations for equilibrium sharing. The research concentrated on the pre-planned formation of the multi-agent system and did not accommodate for a dynamic approach in state space to plan a real time path for the satellites.

M.Pavone at Stanford University working on algorithmic foundations for real time spacecraft motion planning [12], studied the methodology of randomly exploring dense trees to plan in a dynamic environment near these small bodies. In his approach, gravitational influence of bodies was not accommodated. This would affect the developed trajectory, which is one of the key

[1] RRT is not designed to optimize for solution length or any other distance metric.

elements of consideration for our research. An approach like this requires near-real time measurement of gravity mapping and replanning of trajectories in order to execute exploration, fly-bys within the vicinity of bodies in space. With these advancements and the curiosity to explore small bodies in space that host an inherently dynamic environment, it has become the need of the hour to develop path planning approaches for spacecraft to study SSSB during in-orbit maneuvers.

2. Material and methods

The autonomous trajectories used by the spacecraft must account for the gravitational acceleration. The simulation environment relays the location of the satellite, which is used to explore further nodes. The instruments[13] and propulsion systems are assumed to perform to the near-ideal expectations to justify performance viability. This section illustrates the modelling of the gravity field model, algorithm development and the simulation environment for the current research. A decrease in thrust is expected which is synonymous to the motion getting more stable as the on-board field model is refined.

2.1 Gravity field model

Equation 1 gives a generalized foundation to the measurements used for calculating the forces exerted by the SSSB within its sphere of influence. The field models are generated as the potential at a particular distance from the centre of the SSSB with the spacecraft position in the cartesian coordinates. Earth based measurements are considered as the initial model to the motion plan. During actual maneuvers, the field model is updated with real-time measurements, while respecting the previous values. The algorithm identifies the key frames within the field model as illustrated in Table 01.

X	Y	Z	Grav Acceleration (ms^2)
X_0	Y_0	Z_0	G_{a_0}
X_1	Y_1	Z_1	G_{a_1}
.	.	.	.
X_n	Y_n	Z_n	G_{a_n}

Table 01: Key frames for gravity field model: gravitational acceleration at cartesian coordinate based on the in-orbit position of the spacecraft and associated gravitational acceleration.

The algorithm is implemented using the Python programming language and associated libraries mentioned in Appendix B Table 03. The simulation environment is modelled in Visual Python, using the WebGL graphics library and related dependencies. The environment hosts a small solar system body and the exploring spacecraft. The SSSB is a static body, rendered as a mass concentrated model (see figure 04) and exerts the associated gravitational field. The astropy python library engine serves as a source of universal constants and the Sun is considered to be the center of the simulated environment.

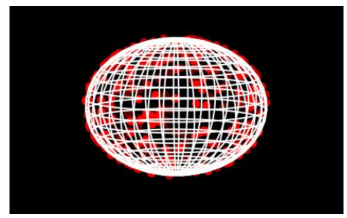

Figure 04: Small solar system body as a mass concentration model

The spacecraft's motion plan solves a single body motion problem. The spacecraft trajectory was visualized in visual python (Figure 05).

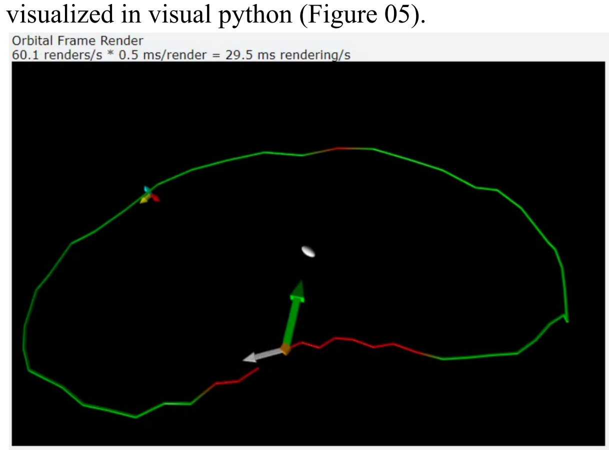

Figure 05: Orbital trajectory(green path) of the spacecraft(orange) in the gravitational field of the SSSB. The direction vector (grey arrow) points towards the heading direction and the gravitational magnitude and attraction is represented by the gravity vector (green arrow)

3. Problem Statement

Our research focuses on active re-planning of spacecraft trajectory by simultaneously measuring and mapping the gravitational forces exerted by the SSSB on the orbiting spacecraft. In the approach, we orbit the spacecraft within the SOI of the SSSB and perform gravity dependant maneuvers by respecting the updated gravity model and planning future maneuvers.

We now formally define the problems that this research has been designed to solve.

Problem 1. Finite time horizon planning for SSSB. Given an SSSB with a gravity model G_f, and a configuration space $C = C_{free} \cup C_{obs}$, and a spacecraft with dynamics D, and a time horizon t, calculate a motion plan $M = < T_1 > .... < T_n >$ such that $M \subset C_{free}$ and respects D.

Problem 2. Minimizing difference between on-board gravity field model and ground truth gravity field. Given an SSSB with a gravity model G_f, and a configuration space $C = C_{free} \cup C_{obs}$, and a spacecraft with dynamics D, and a time horizon t, find the path M that we expect to best refine G_f.

Problem 3. Solving simultaneous motion planning of the spacecraft while continuously updating the gravity field and re-planning the trajectories. Repeatedly, solve problem 2 while simultaneously using the difference between the spacecraft's actual motion from its planned motion to update the gravity model G_f.

3.1 Exploring Viable waypoints to develop the trajectory

We begin by performing motion planning in the known, but inaccurate, gravitational field model acquired from earth-based measurements. The path is then automatically updated to reflect the most recent gravity field model (the gravity model itself continually refined by measuring the forces experienced by spacecraft). We define the configuration space (the search space relevant to motion planning) as the cartesian product of the environment and the spacecraft state space.

The configuration space is denoted by C. Our goal is to find a useful path the free space $C_{free} \subset C$ that maintains orbit while respecting the current gravity model of the SSSB. A trajectory $< T >$ is the geometric curve followed by a spacecraft through C. A feasible trajectory exists entirely in the free space(no collisions occur) and satisfies the motion constraint of the spacecraft. A finite horizon motion plan is created by linking waypoints in C by feasible trajectories. From an abstract graph theoretical point of view, the waypoints and trajectories can be represented as nodes and edges, respectively. In a receding finite time horizon approach, the waypoints (nodes) connected to the motion graph are constrained to exist within a user defined duration into the future.

The trajectory is developed over the optimal way-points (randomly explored within the finite time horizon) which maintain a user defined Euclidean distance from the target body (Figure 06). While nodes are randomly explored using a random sample base approach, only the viable nodes that lie within $H(o_{opt}, r)$, are considered.

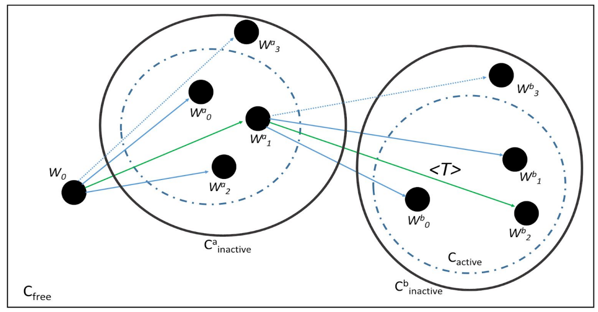

Figure 06: Sparse exploration of waypoints in configuration space where the viable waypoints are selected to explore further and non viable waypoints are stored in inactive space

3.2 Measuring gravitational attraction

The concept of universal gravitation[16] is mathematically expressed as

$$F = -G\frac{m_1 m_2}{r_{21}^2}\hat{r}_{21} \tag{1}$$

where,

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} \tag{2}$$

Work done for the force required for a particular displacement; gravitational potential V at a distance r can be written as

$$V(r) = \frac{1}{m}\int_{\infty}^{r} F.dr \tag{4}$$

using equation (1)

$$V(r) = \frac{1}{m}\int_{\infty}^{r} \frac{-Gm_1 m_2}{r^2} dr \tag{5}$$

on solving the integral, we achieve,

$$V = -\frac{Gm}{r} \tag{6}$$

The gravitational influence experienced by the spacecraft was mapped as a function of its position within the exploring orbit, which was mostly affected arbitrarily by the unknown attractive forces exerted by the target body.

3.3 Simultaneously updating gravity model and re-planning maneuvers

For every new measurement, the motion planner M considers the updated field knowledge G_f to plan the next trajectories for way-point exploration in the configuration space C.

This involves simultaneously mapping of the gravity field during orbital maneuvers by onboard instruments that relay the data to the field model. This simultaneous mapping of the gravity field and planning the next trajectory is analogous to SLAM however it considers the gravity field distribution around the SSSB to decide upon the further trajectories. The planned trajectory however should comply with the control constraints defined in H. The motion plan is developed of a series of trajectories that approach a stable state.

$$M = < T_1 > .... < T_n > \tag{8}$$

Starting with a low-fidelity model, as explorations progress, the gravity model gets more detailed. Simultaneously, the receding horizon tends to a stable orbital motion. Hence the motion plan should have the capability of executing the previous maneuvers with low level knowledge of the gravity field. It should also consider the updated values to execute a series of maneuvers to attain a stable orbit around the SSSB. For every spacecraft position that the gravity is measured, the values are updated on the on-board gravity field model as the latest measurements performed within the orbital maneuvers. The previous field model $\bar{G}_f$ measured over multiple measurements of V is mapped and updated with the real-time measurements G_f.

The orbital maneuvers continue over $< T >$ as explorations progress and a detailed model of the gravity is mapped.

4. Theory and calculation

In our approach, the in-orbit maneuvers are used to explore various state trajectories from the current state to the next-future state, by randomly sampling the nodes within the constraints of distance and velocity. As the spacecraft navigates in free space, a single-body problem is solved considering the spacecraft to be the only moving object. With each iteration, an optimization level is achieved which is not considered as final. As the exploration horizon keeps shifting forward to sample the new space, near-asymptotic solutions are explored within the environment as the posteriori state.

A receding horizon approach for a sampling based motion plan is developed over sparsely expanding trees. It realizes an iterative function over a finite horizon optimization within defined constraints while minimizing the cost. Each exploration is performed for a time horizon in a finite-time interval in the future represented as $[\, t \,;\; t_o + \Delta t \,]$. The control algorithm uses the topology as illustrated in Figure 08.

- Consider the initial field model to execute plan initial trajectory
- Perform real-time measurements update on-board gravity model
- Minimise cost function over control statements
- execute re-planned trajectory based on updated gravity model

A sequence of valid configurations considering the configuration space, free space, target space, obstacle space, is planned, as illustrated in Figure 07. Anything beyond the free space, is believed to be outside the sphere of influence of the gravitational pull of the SSSB and is considered as the danger space, while any space in extreme proximity to the SSSB is the obstacle space.

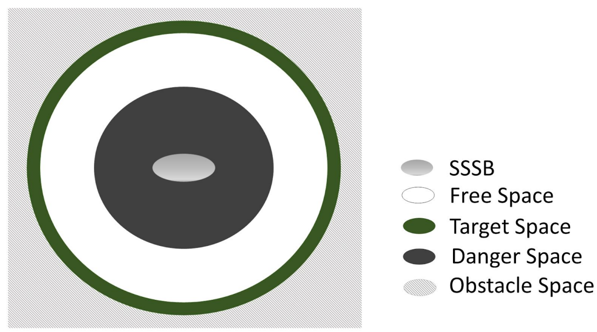

Figure 07 : Motion planning spaces defined for the scope of the research. The spacecraft is planned to explore the gravitational forces exerted by the SSSB and attempt to achieve a stable orbital motion in the target space.

While maneuvering, the gravitational acceleration is measured spatially inwards between the centre of the spacecraft and the SSSB, assuming the SSSB to a continuous mass distribution.

The heuristic defined in equation 9 is a constraint of the spacecraft dynamics (D) and the distance (r) of the spacecraft from the centre of the SSSB within the orbit of exploration.

$$H \,\varepsilon\, f\,[\,D, r\,] \tag{9}$$

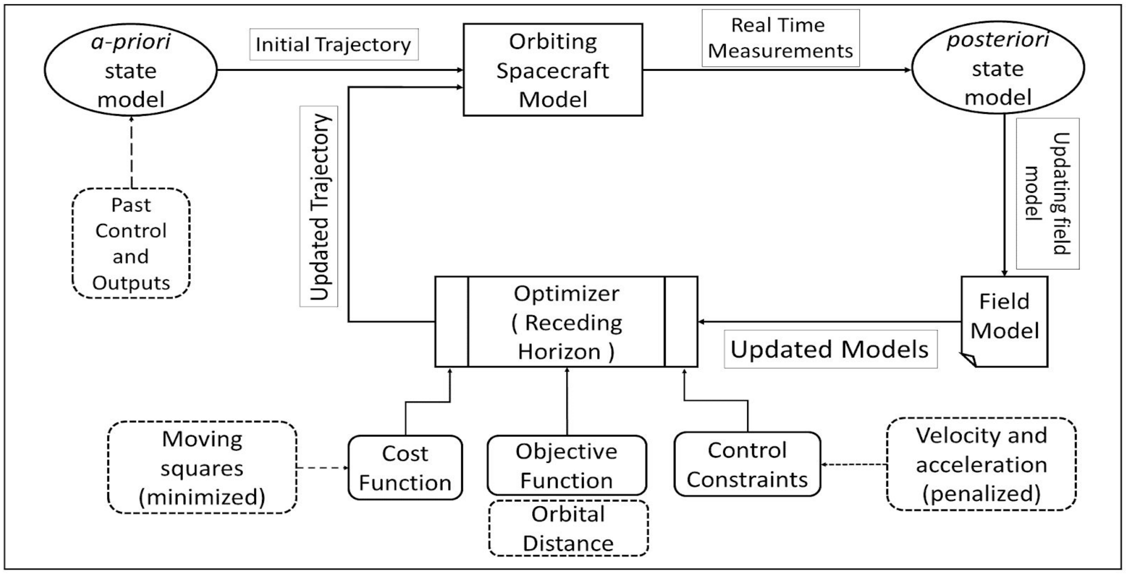

Figure 08 : Trajectory Model Control for spacecraft maneuver and updating trajectory with every epoch. It is modelled as an iterative process control for receding time horizon optimization [15] .

4.1. Cost Function

The cost function in equation 10 was modeled to solve the single body approach for the iterative exploration of nodes while attempting to minimize J over a period of spatial exploration. By considering the difference of the updated gravity model as compared to that of the reference gravity model (ground truth model) and the control nodes are minimised, iteratively..

$$J = \sum_{i=1}^{N} w_{x_i}(\mathrm{R_i} - x_i)^2 + \sum_{i=1}^{N} w_{u_i}\Delta u_i^2 \quad (10)$$

4.2. Objective Function

The objective function is defined as a combination of cost optimization (see equation 10) and a constraint function (see equation 11). This is implemented as an iterative receding horizon approach. During each planning epoch, random sampling is used to explore the space of trajectories within the current finite event horizon. The best trajectory that is found within that planning epoch is used for the next set of spacecraft maneuvers.

The objective function for our approach takes into account the initial point of exploration and is sparse over multiple nodes until the goal is reached. This is denoted as a weighted sum of all iterations over minimizing the cost function.

$$\Lambda = \sum_{i=1}^{N} w_i \Lambda_i \quad (11)$$

The objective function defines the inter nodal distance between two consecutive nodes of exploration, considering the velocity (v) of the spacecraft as -

$$\Lambda_1 = \sum_{i=1}^{N} \sqrt{v_i^2 + v_f^2} \quad (12)$$

where v_i is the velocity at initial node and v_f is the velocity at the goal node. This is penalised by a proximity to obstacle parametric approach with a minimum viable distance determined as

$$\Lambda_2 = \sum_{i=1}^{N} \left(min\left(\frac{dv_i - o_i}{dv_i - o_{near}}\right)\right)^2 \quad (13)$$

4.3. Constraint Function

The constraint function ensures the motion constraints by realizing the obstacle space and the danger space (see figure 06). The motion constraints enforce the spacecraft to maneuver within the limits of the escape velocity for the SSSB bounded between v_{min} and v_{max} with control limit as :

$$v_{i_{min}} < v_i < v_{i_{max}} \quad (14)$$

and the viable distance from the SSSB is reduced as

$$dv < dv_i - o_i < 0 \quad (15)$$

Regarding the nature of the problem statement, the approach needs to be on-board and autonomous.

For the given C that exists in the sphere of influence $C \exists SOI$, the randomly explored way-points need to lie in free space as $O \subset C_{free}$, defined by Λ. The trajectory $< T >$ is then developed over the free space C_{obs} by extending $< T > \rightarrow o_{opt}$ from the current state to the goal state as $o_{init} \multimap o_{goal}$ of edge length dv. The state space for the maneuvers is defined by the objective function to orbit at the desired distance. Every other way-point $o_{rand} \not\in H$ is discarded. The trajectories that have been traversed are forgotten and the cost function J is minimised in order to improve sampling efficiency and trajectory optimization for the succeeding orbital maneuvers. During this maneuver, the gravitational attractions are measured from the start state to the goal state and the on-board gravity model is mapped accordingly, $\bar{G}_f \rightarrow G_f$. The on-board sensors and propulsion are assumed to deliver sufficient performance to execute the motion plan. The approach can be easily modified to include more sensor models, and perform other experiments while establishing dynamic motion planning around the SSSB.

4.4. Algorithm Development

A high level concept of the planner can be seen in Figure 09. Simultaneous mapping of the gravity field is performed as the spacecraft orbits around the target body in the gravitational field of influence. As the distributed gravitational field is mapped onto the on-board gravitational model, the trajectories are replanned to an smoother orbital motion that is optimally traversed as compared to the trajectory in the previous epoch.

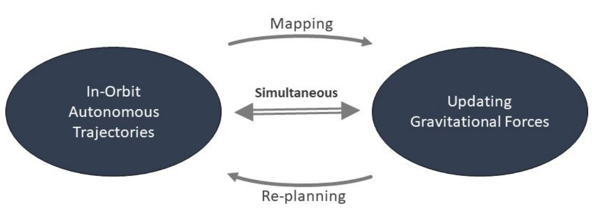

Figure 09: Iterative motion planning for the spacecraft by simultaneously mapping the gravitational influence and replanning further maneuvers.

The motion planner is based on the geometric trees concept of sparse exploration in sample space[4]. The trajectories are developed on the way-points that lie within the heuristics as defined by the control function and the objective function (see sections 4.1,4.2).

However, for algorithmic efficiency in the approach, the geometric trees explore the space sparsely. Only a sparse space is explored during maneuvers, viable way-points are saved, and the rest of way-points and the trajectories are forgotten. The receding horizon method uses the geometric trees in the target space to execute progressice stable maneuvers as the gravity field model becomes more and more refined.

```
Initialize motion planner M ;
Define Orbital Influence SOI ;
Read on-board gravity field model G_f;
while True do
    free space extent C_free
    cost function J(G_f) | H
    while iterations i in C_free do
        while t in range [t_o, t_o + Δt] do
            Sample random nodes o_rand ;
        end
        if o_rand ≘ G_f && o_rand ⊆ H then
            o_rand ↦ Optimal Nodes o_opt ;
                o_opt → way-point list [L] ;
            ō_rand ↦ Near Optimal Nodes o_near ;
                o_near → auxiliary way-point list [L̄] ;
            Develop Trajectory Γ( ⟨T⟩ ⊂ L) ;
                Extend (L_{i-1} ⊸ L_i) ;
                Build Tree(L) ;
            while in L do
                Measure dist(o_exp, o_obs) ;
            end
        else
            Γ( ⟨T⟩ ⊂ L̄) ;
        end
        Measure gravity Ḡ_f at o_exp ;
        minimize dist ;
        return [ Γ, C_free^* , i ] ;
    end
    Update Ḡ_f ↦ G_f;
    forget o_opt and ⟨T⟩;
end
```

Algorithm 01: The algorithm routines simultaneous trajectory planning and gravity mapping of the target body. As the gravity is mapped with each maneuver, the model is updated and the orbital trajectory is replanned and supplied to the motion model, which executes the motion on the developed trajectory

For every way-point, where the gravitational pull exceeds the thrust to keep in the stable orbit, a new waypoint is sampled that causes the spacecraft to move back to the designated distance of exploration. This requires a higher thrust to be exerted by the spacecraft which, as we observe in experiments, decreases with time. According to the trajectory control model in figure 05, every orbit tends to involve smoother motion than the previous orbit. However, such explorations progress with the assumption that the gravity within the sphere of influence is well defined. Deviations between the planned motion and actual motion are then used to help update the gravity model for use in future planning epochs. Thus, it is essential that the results of each planning epoch are computed quickly.

Algorithm 01 illustrates the motion plan algorithm to realize the receding horizon approach for near-optimal exploration of nodes. Every epoch was considered within a time frame of less than 1 minute and individual maneuvers were executed within 0.1 seconds after the plan was generated.

5. Results

The experiment was performed in an orbit, 50 km from the centre of the body. The asteroid exerts a gravitational force within an SOI_{radius} of 350km.

Table 02: Small solar system body specifics

Rationale	Value
Category	Near Earth Asteroid
Mass	6.687e15 kg
Semi Major Axis	1.45 AU km
Radius of SOI	350 km

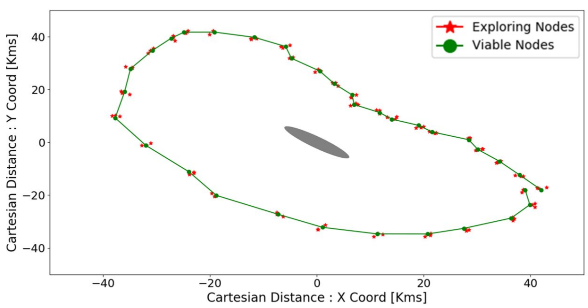

Figure 10: Initial path plan estimate based on earth-based gravity measurement

An initial path plan was developed considering the earth-based gravity measurements as shown in figure 10. It shows that an undulated trajectory was estimated based on the earth-based gravity model. The orbiting spacecraft approached the asteroid with the initial path plan and near-body gravitational accelerations were measured by the on-board gravimeter. This accounted for smaller gravitational accelerations acting on the body. The simulation, explored for about 1500 points, ran for 3 hours 12 minutes to reach a stable orbit, as the onboard gravity field progressively refined during motion (see figure 01). Figure 11 shows the improvement of the trajectory(on the left) with the gravity model (on the right) continuously updating and accommodating smaller forces during the spacecraft motion. Starting from a disturbed orbital trajectory based on the initial gravity model, the spacecraft attained a stable orbital motion as the gravity model got refined.

The simulation provided visuals of the gradual stabilization of the orbital maneuvers[2]. A visually comparative plot of the earth-based acceleration model and the in-orbit acceleration model were plotted as a digital elevation model as shown in figure 12. The figure on the right, shows a high fidelity model as compared to the initial gravity model with which the spacecraft approached the SSSB.

The detailed model shows that smaller forces were also accommodated during the maneuvers in the vicinity of the SSSB that were otherwise not mapped by earth-based observations.

With every epoch the thrust exerted by the spacecraft was measured. This thrust was exerted to counteract the gravitational forces experienced by the spacecraft while performing orbital maneuvers. The spacecraft tackled high gravitational pull, by thrusting away from the centre of the SSSB and on the contrary exerted a thrust towards the centre of the SSSB; in either cases working actively to maintain the desired distance of exploration from the SSSB.

The corresponding thrust values from the engine were observed to have decreased as orbital explorations increased. This was synonymous to the gravitational model getting more and more known and being updated simultaneously. As the new model was respected, lesser impulsive thrust was needed to maneuver the spacecraft within the orbit of exploration. Figure 13 shows decreasing thrust values as compared from the initial exploration as it increased. Figure 14 (zoomed in) shows a logarithmic decreasing trend of the thrust exerted by the spacecraft.

[2] A simulation of the spacecraft trajectory around the SSSB in the gravitational field of influence with smoothening of the orbital motion can be seen here - **Spacecraft Trajectory**. The simulation for the gravity update as shown in Figure 01 can be seen here - **Gravity Model Update**.

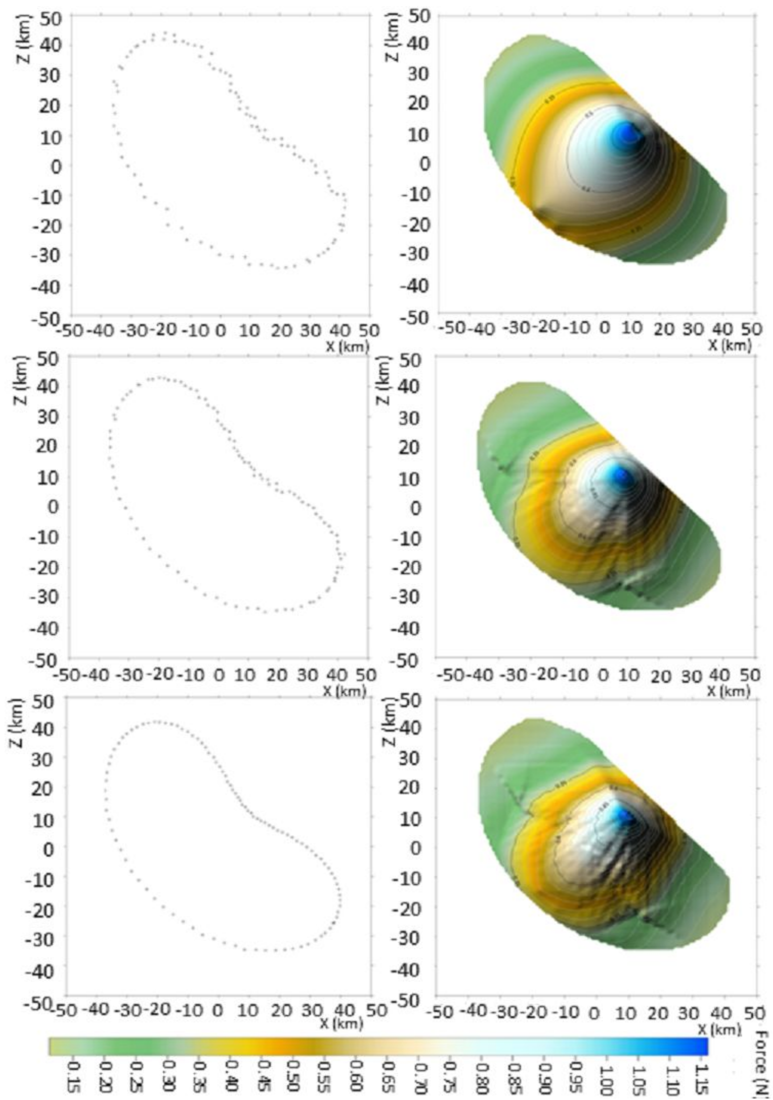

Figure 11 : Trajectory points for spacecraft motion at different explorations points within an orbit of 50 km. The trajectory can be seen to get more refined and stable as the gravity model got more details with succeeding explorations.

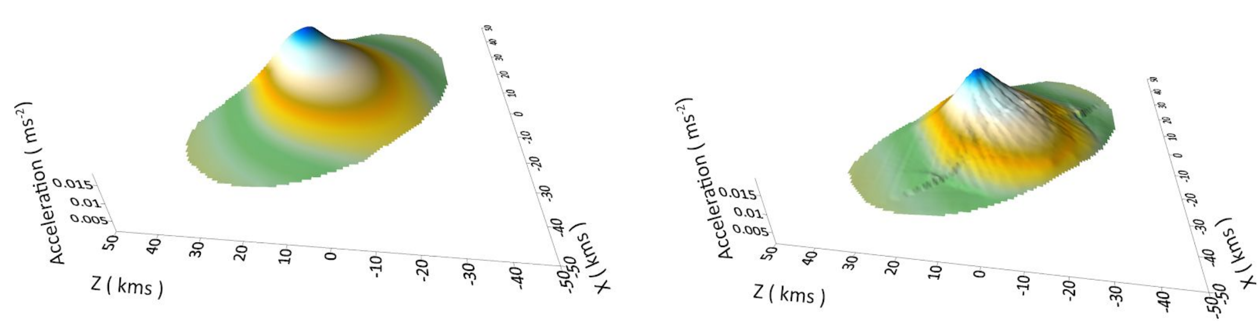

Figure 12 : Visual Comparison of Earth-based (left) and In-Orbit (right) gravitational acceleration model after 7 orbital maneuvers at a distance of 50 km.

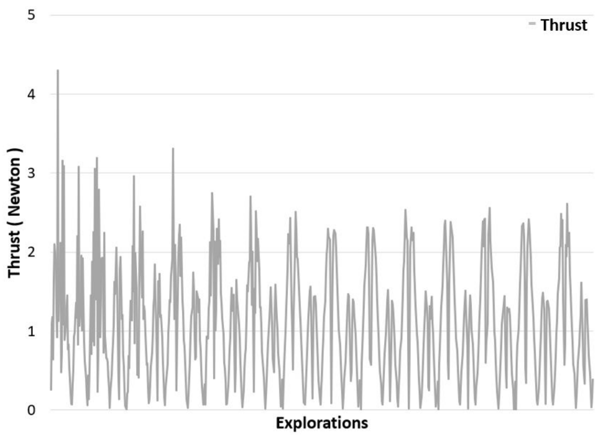

Figure 13 : Decreased thrusting with progressive orbital explorations as spacecraft approaches stable maneuvers according to the updated gravitational field.

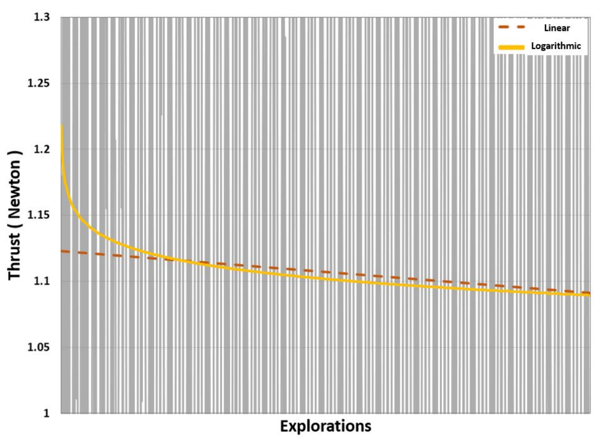

Figure 14 : Decreasing thrust trend-line showing that as explorations progressed the gravity field model was better known, therefore smoother trajectories were executed which required lesser impulsive thrusting for the maneuvers.

6. Discussion

Starting from the initial trajectory as shown in figure 10, the succeeding trajectories in figure 11 show that the spacecraft eventually attains a stable orbit. The updating gravity models plotted, in figure 01 show the constant updating of the field measurements to the on-board model. The dip in the orbit of exploration illustrates that the spacecraft experienced higher attraction and hence is pulled towards the centre of the SSSB. This portion of the motion plan requires denser way-point exploration for the spacecraft to return to the safe orbit of exploration. With each maneuver as the gravity model is 'well-informed' of the highly attractive gravitational force in that particular space, the trajectory is corrected, leading to smoother and smoother motion plans. The values from this set of experiments are recorded over the time span of 1500 explorations. After 1500 the spacecraft is observed to perform stable maneuvers, which we attribute to the gravity field becoming 'well known'. A visual comparison between the earth-based measurements and the in-orbit measurements in figure 12 can be seen, as we achieve a higher fidelity model after the explorations, that map larger forces of attraction while accounting for the smaller forces as well. A thrust reduction of 3.06% was observed, that less thrust was exerted by the spacecraft as the maneuvers became more stable. This is synonymous to the gravity profile getting more detailed. Locations where the gravitational pull was higher (considering the dip in the orbit), would be a favorable site study for the material composition of the asteroid and to plan landing missions.

7. Conclusions

A novel motion planning approach is demonstrated to realise dynamic and autonomous orbital exploration around small solar system bodies using the arbitrary gravitational forces exerted by the body. The experimental outcomes discussed in this thesis, showcase the feasibility of the motion planning algorithm as an efficient approach to execute safe maneuvers in the gravitational field and obtain fine-grained field models. The stable orbital trajectory achieved within the sphere of influence proves the competence of using a receding horizon approach.

We believe that this demonstrates that while earth-based observations may be adequate to reach the SSSB, they may not be sufficiently 'well-informed' to perform pre-calculated orbital maneuvers in the vicinity of SSSB. Such maneuvers are arguably necessary to study SSSB. Therefore, algorithms that combine dynamic path planning with gravity field refinement such as those presented in the current paper, will likely play a crucial role in future missions.

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Appendix A (Sphere of Gravitational Influence)

Every body in the universe exerts an attractive force on every other body due to the gravitational force of attraction. However, the magnitude of the force is only relevant to motion planning within a specific region around the body (overpowered by other forces at a greater distance). This region of gravitational influence for a particular body is termed as Sphere of Influence. In terms of conic approximation, the SOI is generally referred to as the boundary that causes a change in the trajectory of the orbiting body.

$$SOI_{radius} \approx a\left(\frac{m_1}{m_2}\right)^{0.4} \tag{16}$$

where m_1 is the orbiting body and m_2 is the body being orbited around

Appendix B (Software and Hardware Libraries)

Table 03: Description of the Operating systems used

Operating System	Version	Description
Ubuntu	16.04 LTS	Operating System
Microsoft Windows	10 Home	Operating System

Table 04: Description of the Languages and Libraries used

Languages and Libraries	Version	Description
Python	3.7	Language
WebGL	1.0	Visualization
AstroPy	4.0.1	Physics Constants
SciPy	1.4.1	Mathematics

Table 05: Description of the Languages and Libraries used

Hardware	Specifications
Processor	Intel(R) Core^{TM} i5-8250U
Graphics Card	Intel UHD Graphics 620
CPU Frequency	1.60 GHz

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